



Reg. No. : .....

Name : .....

**First Semester M.Tech. Degree Examination, April 2016**  
**(2013 Scheme)**  
**Branch : Mechanical Engineering**  
**Streams : Thermal Engineering and Propulsion Engineering**  
**MMA 1001 : APPLIED MATHEMATICS**

Time : 3 Hours

Max. Marks : 60

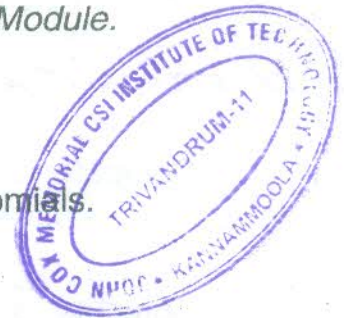
- Instructions :** 1) Answer **any two** questions from **each** Module.  
 2) **All** questions carry **equal** marks.

## MODULE - I

1. a) State and prove Rodrigue's formula for Legendre polynomials.  
 b) Show that  $\int_{-1}^1 P_m(x)P_n(x)dx = 0$  if  $m \neq n$ .
2. a) Solve  $x \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{1}{4}y = 0$  in terms of Bessel functions.  
 b) When  $n$  is an integer, prove that  $J_{-n}(x) = (-1)^n J_n(x)$ .
3. Solve using Laplace Transform method the boundary value problem  
 $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ ;  $0 < x < 2$ ;  $t > 0$   $y(0, t) = 0$ ,  $y(2, t) = 0$ ,  $y(x, 0) = 3 \sin(2\pi x)$

## MODULE - II

4. a) Find the curve on which the functional  $\int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 2xy) dx$  with  $y(0) = 0$  and  $y\left(\frac{\pi}{2}\right) = -1$  is an extremum.  
 b) Find the extremal of the functional  $\int_0^{\pi/2} ((y')^2 + (z')^2 + 2yz) dx$  given that  $y(0) = 0$ ,  $y(\pi/2) = -1$ ,  $z(0) = 0$ ,  $z(\pi/2) = 1$ .





5. a) Find the differential equation corresponding to

$$y(x) + 3 \int_0^x (x-t)^2 y(t) dt = x^2 - 3x + 4$$

- b) Show that  $y(x) = \frac{1}{\pi\sqrt{x}}$  is a solution of  $\int_0^x \frac{y(t)}{\sqrt{(x-t)}} dt = 1$ .

6. a) Show that  $\left(\hat{\hat{z}}\right)^y = z$ , where  $\left(\hat{z}\right)$  and  $\left(\check{z}\right)$  denote the DFT and IDFT of  $z$ .

- b) Let  $z = (1, i, 2 + i, -3)$

i) Compute  $\left(\hat{z}\right)$

- ii) Compute  $\left(\check{z}\right)^y$  directly and check that you get  $z$ .

### MODULE - III

7. a) Define span of a set of vectors. Is the vector  $v = (2, -5, 3) \in \mathbb{R}^3$  an element in the span of  $v_1 = (1, -3, 2)$ ;  $v_2 = (2, -4, -1)$ ;  $v_3 = (1, -5, 7)$ .
- b) Define basis and dimension of a vector space. Show that the dimension of the vector space  $W = \text{span} \{v_1 = (1, -2, 1), v_2 = (1, 1, 1)\}$ .
8. a)  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  is defined by  $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$  show that  $T$  is linear. Also find the kernel of  $T$ .
- b)  $B_1 = \{u_1 = (1, -2), u_2 = (3, -4)\}$  and  $B_2 = \{v_1 = (1, 3), v_2 = (3, 8)\}$  are bases of  $\mathbb{R}^2$ . Find the change of basis matrix from  $B_1$  to  $B_2$ .

9. Show that  $B = \left\{ v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$  is a basis of  $\mathbb{R}^3$ . Is the basis  $B$

orthonormal? Why? If the basis  $B$  is not orthonormal, construct an orthonormal basis using  $B$ .