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Reg. No. : ....

Name : ....

First Semester M.Tech. Degree Examination, April 2016
(2013 Scheme)
Branch : Mechanical Engineering
Streams : Thermal Engineering and Propulsion Engineering
MMA 1001 : APPLIED MATHEMATICS

Time : 3 Hours Max. Marks : 60

Instructions : 1) *Answer* ***any two*** *questions from* ***each*** *Module.*
2) ***All*** *questions carry* ***equal*** *marks.*

MODULE – I

1. a) State and prove Rodrigue's formula for Legendre polynomials.

   b) Show that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$ if $m \neq n$.

2. a) Solve $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{1}{4}y = 0$ in terms of Bessel functions.

   b) When n is an integer, prove that $J_{-n}(x) = (-1)^n J_n(x)$.

3. Solve using Laplace Transform method the boundary value problem
   $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial t^2}$; $0 < x < 2$; $t > 0$ $y(0, t) = 0$, $y(2, t) = 0$, $y(x, 0) = 3\sin(2\pi x)$

MODULE – II

4. a) Find the curve on which the functional $\int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 2xy\right)dx$ with $y(0) = 0$ and $y'\left(\frac{\pi}{2}\right) = -1$ is an extremum.

   b) Find the extremal of the functional $\int_0^{\pi/2} \left((y')^2 + (z')^2 + 2yz\right)dx$ given that $y(0) = 0$, $y(\pi/2) = -1$, $z(0) = 0$, $z(\pi/2) = 1$.

P.T.O.

5. a) Find the differential equation corresponding to

$$y(x)+3\int_0^x (x-t)^2 y(t)\,dt = x^2-3x+4$$

b) Show that $y(x)=\dfrac{1}{\pi\sqrt{x}}$ is a solution of $\displaystyle\int_0^x \frac{y(t)}{\sqrt{(x-t)}}\,dt = 1$.

6. a) Show that $\left(\hat{z}\right)^{\vee} = z$, where $\left(\hat{z}\right)$ and $\left(\check{z}\right)$ denote the DFT and IDFT of z.

b) Let z = (1, i, 2 + i, –3)

i) Compute $\left(\hat{z}\right)$.

ii) Compute $\left(\hat{z}\right)^{\vee}$ directly and check that you get z.

MODULE – III

7. a) Define span of a set of vectors. Is the vector $v = (2, -5, 3) \in R^3$ an element in the span of $v_1 = (1, -3, 2)$; $v_2 = (2, -4, -1)$; $v_3 = (1, -5, 7)$.

b) Define basis and dimension of a vector space. Show that the dimension of the vector space $W = \text{span}\,\{v_1 = (1, -2, 1),\ v_2 = (1, 1, 1)\}$.

8. a) $T : R^3 \to R^3$ is defined by $T(x, y, z) = (x + 2y - z,\ y + z,\ x + y - 2z)$ show that T is linear. Also find the kernel of T.

b) $B_1 = \{u_1 = (1, -2),\ u_2 = (3, -4)\}$ and $B_2 = \{v_1 = (1, 3),\ v_2 = (3, 8)\}$ are bases of R^2. Find the change of basis matrix from B_1 to B_2.

9. Show that $B = \left\{ v_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, v_2 = \begin{bmatrix}1\\1\\-1\end{bmatrix}, v_3 = \begin{bmatrix}1\\0\\1\end{bmatrix} \right\}$ is a basis of R^3. Is the basis B orthonormal ? Why ? If the basis B is not orthonormal, construct an orthonormal basis using B.

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